Evaluating spatial quantitative precipitation forecasts in the form of binary images

Merging and Matching with the Baddeley Delta Metric

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Outline

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• Baddeley Delta Metric
• Merging and Matching Strategy
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Motivation: Verification of Quantitative Precipitation Forecasts

Forecast

Observations
Motivation: Objectification Approach

Forecast

Observations
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Example

- First four forecasts have POD=0; FAR=1; CSI=0
  - i.e., all are equally “BAD”
- Fifth forecast has POD>0, FAR<1, CSI>1
- Traditional verification approach identifies “worst” forecast as the “best”
The goal is to find the best mergings and matchings.

- We need a metric.
- Using the chosen metric, we need a reasonably fast strategy for merging and matching.
- Baddeley metric is designed for the purpose of comparing images, and it can be fast.
The Baddeley delta metric is essentially an average of shortest distances between every pixel in an image raster and a set.
Baddeley Delta Metric

For a raster of pixels, $X$, the Baddeley delta metric for comparing set $A \subseteq X$ to set $B \subseteq X$ ($\Delta^p_w(A, B)$) is:

$$\Delta^p_w(A, B) = \Delta = \left[ \frac{1}{n(X)} \sum_{x \in X} |w(d(x, A)) - w(d(x, B))|^p \right]^{1/p},$$

where $d(x, A)$ is the shortest distance from a point $x \in X$ to the set (object) $A$, $1 \leq p < \infty$ and $w$ is a concave function ($w(s + t) \leq w(s) + w(t)$) that is strictly increasing at zero ($w(t) = 0$ iff $t = 0$).
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We use $p = 2$ and $w(t) = \min(t, 100)$

Lower values of $\Delta$ mean sets are more similar to each other.
Merging and Matching Strategy

Given a forecast image object with \( n_f \) objects and an analysis image object with \( n_a \) objects.

- Which objects from one field match “best” with objects from the other field?
- Which objects within an image should be merged?
- Ideally, one would compute all \( 2^{n_f} \cdot 2^{n_a} \) \( \Delta \)'s for all possible mergings. Too computationally intensive!
- Here, we propose looking at a reasonable subset of the possible mergings.
Merging and Matching Strategy

Let $i = 1, \ldots, n_f$ denote the $i^{\text{th}}$ forecast object, and $j = 1, \ldots, n_a$ the $j^{\text{th}}$ analysis object.

1. Create the matrix $[\Delta(i, j)]$
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1. Create the matrix \([\Delta(i, j)]\)

2. Rank the values from Step 1.
Let $i = 1, \ldots, n_f$ denote the $i^{th}$ forecast object, and $j = 1, \ldots, n_a$ the $j^{th}$ analysis object.

1. Create the matrix $[\Delta(i, j)]$

2. Rank the values from Step 1.
   
   For each object $i$, let $j(1), \ldots, j(n_a)$ denote the objects with lowest to highest $\Delta(i, j)$ (and vice-versa)
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3. Create a matrix with $\Delta(i, j(1)), \Delta(i, j(1, 2)), \ldots, \Delta(i, j(1, \ldots, n_a))$ Do the same for the other direction. (i.e., $\Delta(j, i(1)), \ldots, \Delta(j, i(1, \ldots, n_a))$)
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4. Merge and match objects by comparing the above three matrices.

5. Accept merges/matches only for $\Delta$ below a chosen threshold.
Test Case 1

<table>
<thead>
<tr>
<th>Forecast</th>
<th>Observations</th>
<th>Δ</th>
</tr>
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<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.06</td>
</tr>
<tr>
<td>7</td>
<td>6, 7</td>
<td>0.07</td>
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<tr>
<td>1, 2, 4</td>
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<td>0.08</td>
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<tr>
<td>8</td>
<td>5</td>
<td>0.08</td>
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</table>
Test Case 2

<table>
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<tr>
<th>Forecast</th>
<th>Observations</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1, 2, 4</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.29</td>
</tr>
</tbody>
</table>
Summary and Ongoing Work

- Difficult to perform verification on QPF.
- One way to solve the problem is to objectify the QPF, and analyze the “cleaner” resulting objects.
- Before verification can be done on the resulting objects, they must be matched/merged.
- Baddeley delta metric is useful for comparing images.
- Need to compare our strategy with other approaches (e.g., fuzzy logic).
- Adapt our strategy so that the same (merged) analysis objects are compared to different forecasts.