The Spatial Prediction Comparison Test (SPCT)

WRF Users’ Workshop
28 June 2013

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G. 2013, MWR, 141 (1), 340 - 355
Mean $\approx 2.31$ mm/h
Std. Err. $\approx 0.0095$ mm/h

Mean $\approx 2.34$ mm/h
Std. Err. $\approx 0.0092$ mm/h

High Spatial Correlation! Std. Err.’s are too small!

Ignoring grid points where differences are zero.
High Spatial Correlation! Std. Err.’s are too small!

Mean ≈ 0.13 mm/h
Std. Err. ≈ 4.716 mm/h

Mean ≈ -0.41 mm/h
Std. Err. ≈ 4.762 mm/h

Mean ≈ -0.52 mm/h
Std. Err. ≈ 4.878 mm/h

Ignoring grid points where differences are zero.
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Some Notation

\[ d_1 = d(F_1, O) = \text{loss}(F_1, O) \quad \quad d_2 = d(F_2, O) = \text{loss}(F_2, O) \]

(e.g., \( d_1 = |F_1 - O| \))

Note: \( d_1 \) is calculated at each grid point, so that each grid point has a value that is ultimately aggregated over the entire field.

\[ D = d_1 - d_2 \]

Note: \( D \) is a spatial field, called the loss differential field, of the difference in loss function values (again at each grid point) for \( F_1 \) and \( F_2 \).

\[ \overline{D} = \frac{1}{N} \sum_{i \in (x,y)} D_i \]

Average loss differential

Precipitation: calculate only over grid points where at least one field is non-zero.
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Test Statistic

$$S_D = \frac{\bar{D} - \mu}{\text{Var}(\bar{D})} = \frac{\bar{D}}{\text{Var}(\bar{D})}$$

Null hypothesis is that the test statistic is zero. Two-sided alternative is that it is not equal to zero. Can also perform a one-sided less (or greater) than alternative instead.

Assumption for test is that the test statistic follows a standard normal distribution. That is, that the mean loss differential is normally distributed.

The estimation of

$$\text{Var}(\bar{D})$$

is the trick.
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$(1 - \alpha) 100\%$ Confidence Intervals can also be estimated using:

$$\bar{D} \pm z_{\alpha/2} \sqrt{\text{Var}(\bar{D})}$$
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\[
\hat{\text{Var}}(\bar{D}) = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \hat{C}(h_{ij}) \\
h_{ij} \text{ are all points separated by the amount } |i - j|.
\]

\[
\hat{C}(h_{ij}) = \hat{\gamma}(\infty) - \hat{\gamma}(h_{ij})
\]

\[
\gamma(h_{ij}) = \frac{1}{2|N(h_{ij})|} \sum_{N(h_{ij})} \left[ D(x_i, y_i) - D(x_j, y_j) \right]^2
\]

A parametric variogram is fit to the empirical one, and it is the sums of the lags of the parametric model that are used to estimate the variance of the mean loss differential.
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p-value for one-sided less than hypothesis is about 0.479 (fail to reject null hypothesis that mean loss differential is equal to zero).

Mean loss differential is about -0.21 with 95% CI about (-8.12, 7.69)
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Examples of Loss Functions:

- Simple loss: \( F - O \)
- Square Error Loss: \((F - O)^2\)
- Absolute Error Loss: \(|F - O|\)
- Correlation Skill

\[(O - \bar{O})(F - \bar{F})\]
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Summary and Conclusions

- Test is for competing forecasts.
- Assumption is that the test statistic follows a standard normal distribution.
- No assumption about the distributions of O, F1 or F2.
- Works for any loss function.
- Accounts for spatial correlation.
- Does not require a gridded spatial field.
- Computationally efficient for gridded spatial fields and other spatial fields that do not have a lot of locations.
- Cannot always find a good fitting variogram.
- Does not account for location errors or accumulation of numerous small-scale errors.
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Warping + Absolute Error Loss
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Warping + Absolute Error Loss

\[
\text{loss}\left(F(W(x_i, y_i)), O(x_i, y_i)\right) + \text{loss}\left((x_i, y_i), W(x_i, y_i)\right)
\]

\[
\left|F(W(x_i, y_i)) - O(x_i, y_i)\right| + \sqrt{x_i - W_x(x_i, y_i)^2 + (y_i - W_y(x_i, y_i))^2}
\]

\(W\) represents a vector-valued (warping) function that yields the new coordinates of the points after applying the (optimal) warp.
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Warping + Absolute Error Loss
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Warping + Absolute Error Loss

Fig. 3. Results for (top) mean differential $\bar{D}$ based on warping plus AE loss, and (bottom) the associated test statistic. Dotted lines indicate continuously available scores. Negative (positive) values imply that ARW-WRF (NMM) is better on average in terms of warping loss.

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Summary and Conclusions

• Applying image warping first results in a test that accounts for location errors as well as spatial correlation.
• Optimizing the warp function takes time, but is not terribly inefficient either.
• Can be applied to non-gridded fields, but perhaps trickier.
• R image warping package on its way.

Future Work

Additional uncertainty introduced because of uncertainty associated with fitting the warp function to the fields. Can this be incorporated into the test?

It is possible to extend this to a test for spatio-temporal fields, but how exactly?
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SpatialVx: R package for implementing spatial forecast verification methods. Available on CRAN, but still incomplete and not heavily tested.

http://www.ral.ucar.edu/projects/icp/SpatialVx/

Includes functions for implementing the spatial prediction comparison test. See help files for spatMLD, fit.spatMLD as well as their summary and plot method functions. Image warping + AE loss will be included once the warping package is finished and uploaded.