CORRESPONDENCE

Comments on "Revisiting the Relationship between Eyewall Contraction and Intensification"

CHANH Q. KIEU

Department of Earth and Atmospheric Sciences, Indiana University Bloomington, Bloomington, Indiana

DA-LIN ZHANG

Department of Atmospheric and Oceanic Science, University of Maryland, College Park, College Park, Maryland

(Manuscript received 11 January 2017, in final form 14 August 2017)

ABSTRACT

This comment presents some concerns with the study of Stern et al. and their misinterpretation of the contraction of the radius of the maximum wind (RMW) in tropical cyclones. It is shown that their geometrical RMW contraction model provides little dynamical understanding of the RMW contraction during tropical cyclone intensification, and it differs fundamentally from the RMW contraction model of Willoughby et al. that was derived from the directional derivative concept. Moreover, it is demonstrated that Stern et al. were mistaken in commenting on the derivation of the governing equation for the RMW contraction in Kieu.

1. Introduction

Recently, Stern et al. (2015, hereinafter S15) examined the contraction of the radius of maximum wind (RMW) during tropical cyclone (TC) intensification. A central issue raised by S15 is the implicit perception that the intensification of the maximum tangential wind (V_{MAX}) is always concurrent with the eyewall contraction during TC rapid intensification. Using an idealized simulation and observational analyses, S15 pointed out that such simultaneity between the contraction of the RMW and the intensification of V_{MAX} is not always valid, as the RMW could quickly slow down during TC rapid intensification despite continuing increases in V_{MAX}.

While S15's study of this relationship between the eyewall contraction and intensification is consistent with previous observational and modeling studies (e.g., Corbosiero et al. 2005; Xu and Wang 2010; Kieu 2012, hereinafter K12), S15's approach to the understanding of the RMW contraction contains some inherent issues that render their interpretation problematic. In addition, S15 misinterpreted K12's derivation of an equation for the RMW contraction in their discussion. In section 2, we summarize S15's main results and argue that S15's interpretation of the RMW contraction based on a geometrical approach provides little understanding of the processes underlying the RMW contraction. In section 3, we discuss S15's misinterpretation of K12's work on the derivation of the governing equation for the contraction of the RMW during TC intensification.

2. A geometrical interpretation of the RMW contraction

S15 started their analysis of the RMW contraction by considering first a radial tangential wind profile $V = V(r, t)$ with the peak value at some radius, presumably the RMW, such that

$$\left.\frac{\partial V(r,t)}{\partial r}\right|_{r=R} = 0, \qquad (1)$$

where R denotes the RMW and the uppercase variable $V(r, t)$ denotes the tangential wind evaluated at $r = R$. After taking a time derivative of Eq. (1), they obtained

$$\frac{\partial}{\partial t}\left(\frac{\partial V}{\partial r}\right) + \frac{\partial}{\partial r}\left(\frac{\partial V}{\partial r}\right)\frac{dR}{dt} = 0. \qquad (2)$$

Corresponding author: Dr. Chanh Kieu, ckieu@indiana.edu

DOI: 10.1175/JAS-D-17-0011.1

Rearranging Eq. (2), S15 arrived at an equation that describes how the RMW is changing with time as follows:

$$\frac{dR}{dt} = -\frac{(\partial/\partial r)(\partial V/\partial t)}{\partial^2 V/\partial r^2}\bigg|_{\mathrm{RMW}}, \tag{3}$$

which is Eq. (7) in S15. In S15, the numerator $(\partial/\partial r)(\partial V/\partial t)$ is coined by a term ''the radial gradient of the time tendency of V,'' and the denominator $\partial^2 V/\partial r^2$ is called ''curvature or 'sharpness' of the radial profile of V'' (S15, p. 1288). S15 then treated these two terms as some type of physical mechanisms that could explain the contraction of the RMW. Their detailed analyses of these two terms (see Figs. 5 and 6 in S15 and the associated discussion therein) led to their key conclusion that the RMW contraction is due to the increased radial gradients of the wind tendency in the numerator of Eq. (3) but not to the wind curvature term in the denominator. Although there is no mathematical issue in their derivation of Eq. (3), we have several concerns, to be discussed below, with the use of this equation to provide an understanding of the RMW contraction.

First, one may notice that Eq. (3) is purely geometrical with no dynamical content and can be applied to any radial wind profile $v(r, t)$ that possesses a maximum or minimum. In fact, one could use an arbitrary smooth profile $v(r, t)$ with one or multiple maxima to arrive at the same equation as Eq. (3) for all points r^* where $v(r^*, t)$ is a local extreme with $\partial v/\partial r|_{r=r^*} = 0$. In addition, S15's contraction equation [Eq. (3)] requires that both $\partial^2 V/\partial r\partial t$ and $\partial^2 V/\partial r^2$ have to be given before calculating the contraction rate. It just happens a priori from the model output of an axisymmetric vortex that the denominator and the numerator of Eq. (3) have a radial profile as shown in their Figs. 5 and 6. Namely, the radial distributions of either $\partial^2 V/\partial r^2$ or $\partial^2 V/\partial r\partial t$ in S15's Figs. 5 and 6 are not derived from the prognostic equation for the RMW contraction but are given in advance. In this sense, using these given distributions to explain the RMW contraction provides little understanding into the processes leading to RMW contraction, because of the following critical question: Why could such radial distributions of the tangential wind develop? Specifically, one would like to know what physical mechanisms can explain the radial distributions shown in their Figs. 5 and 6 during the RMW contraction, not a priori assuming these distributions and then plugging into the geometrical relationship Eq. (3) to see how the contraction takes place.

Second, it should be mentioned that for all practical purposes, has one known $v(r, t)$ and $\partial v(r, t)/\partial t$ in advance, which are needed to compute the denominator and numerator of Eq. (3), one would know immediately where the RMW will be and what the contraction rate would be in the next without any reference to Eq. (3). Stating that the contraction of the RMW is ''a result of an increase in the radial gradient of the wind tendency'' (S15, p. 1296) or ''contraction is halted in association with a rapid increase in the sharpness of the tangential wind profile'' (S15, p. 1283) is similar to a statement that a change of velocity is a result of an acceleration. There is nothing wrong with such a statement, but such a statement bears little relation to physical processes that we are aiming to understand, which are *the physical forcings that produce such an acceleration*. This point is highlighted here because the geometrical approach presented in S15 in this sense contains no dynamical explanation for the RMW contraction.

Third, unlike S15 mentioned, Eq. (3) has no clear similarity to the RMW contraction equation presented in Willoughby et al. (1982, hereinafter W82), which is supposedly Eq. (3) in the original work of W82. An examination of the original Eq. (3) in W82 shows that W82 appeared to derive their RMW contraction equation from Petterssen's (1956) definition of ''a derivative in a moving frame of reference'' (or the directional derivative) that contains no second-order derivative in the denominator (Petterssen 1956, sections 3.2 and 3.3). Indeed, the principle underlying W82's Eq. (3) can be seen from the definition of the directional derivative along a parameterized curve in the cylindrical coordinate (r, θ, z) as follows (e.g., Schutz 1980):

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \dot{r}\frac{\partial v}{\partial r} + \dot{\theta}\frac{\partial v}{\partial \theta} + \dot{z}\frac{\partial v}{\partial z}, \tag{4}$$

where $v(r, \theta, z)$ is the Eulerian field of the tangential wind and the dot denotes the time derivative along a given curve.[1] If we can (i) follow the RMW point such that dv/dt can be computed along the way; (ii) express the RMW contraction rate, $c = -\dot{r} = -dR/dt$, as defined by W82, as we follow the curve $R(t)$ traced out by the RMW in the (r, z) plane; and (iii) assume that the contraction is along the inflow such that vertical motion is negligible, then c can be derived from Eq. (4) as follows:

$$c = -\frac{dR}{dt} = \frac{\left(\dfrac{\partial v}{\partial t}\right)_{\max} - \left(\dfrac{dV}{dt}\right)_{\mathrm{RMW}}}{\partial v/\partial r}, \tag{5}$$

where $(\partial v/\partial t)_{\max}$ is the maximum tendency of the tangential wind at a fixed radius (e.g., at the RMW at the

[1] We emphasize that Eq. (4) is the definition of a directional derivative along a parameterized curve. By convention, we always assume that such a parameterized curve can be defined as a smooth horizontal curve $R(t)$ traced out by the RMW in the (r, z) plane.

current time step) and $(dV/dt)_{\mathrm{RMW}}$ is the differenced V_{MAX} at two different (e.g., the next and current) time steps at two different RMWs that may not be the same during contraction. Assuming that one has two radial profiles for the tangential wind $v(r, z_0, t_1)$ at time t_1, and $v(r, z_0, t_2)$ at time t_2 obtained, for example, from aircraft observations at a fixed flight level z_0 as discussed in W82, then all the right-hand side (rhs) terms of Eq. (5) can be readily computed, thus allowing us to quantify the contracting rate c.

To illustrate how the directional derivative in Eq. (4) can lead to the RMW contraction rate as given by Eq. (5), Fig. 1 shows a snapshot of the tangential wind profile at one specific height level at two different times t and $t + \Delta t$ with a note that in the limit of $\Delta t \rightarrow 0$,

$$\left(\frac{dV}{dt}\right)_{\mathrm{RMW}} \approx \frac{V_B - V_A}{\Delta t} = \frac{V_B - V_C}{\Delta t} + \frac{V_C - V_A}{\Delta r}\frac{\Delta r}{\Delta t} \approx \left(\frac{\partial v}{\partial t}\right)_{\max} - c\frac{\partial v}{\partial r}. \qquad (6a)$$

A rearrangement of Eq. (6a) leads to Eq. (5), thus justifying the basic directional derivative principle in W82's equation for the RMW contraction. It is apparent from Fig. 1 that if we can trace the RMW to obtain $(dV/dt)_{\mathrm{RMW}}$, and if we know the maximum local change of v, that is, $(dv/dt)_{\max}$ from the flight-level data as presented in W82, then either Eq. (5) or Eq. (6a) gives an estimation of the RMW contraction rate. As shown above, Eq. (5) or Eq. (6a) derived from the directional derivative principle at the RMW has no connection to the second-order derivative of tangential wind profile, that is, $\partial^2 V/\partial r^2$ as appeared in the denominator of Eq. (3) obtained by S15.

Unlike the claim in S15, we are not able to derive Eq. (5) from Eq. (3) above, which is Eq. (7) in S15. It appears that S15 incorrectly replaced the total derivative $(dV/dt)_{\mathrm{RMW}}$ in the original equation in W82 by a partial time derivative $\partial V/\partial t$ in the numerator of their Eq. (8). This replacement is not correct because $(dV/dt)_{\mathrm{RMW}}$ in W82 is not the total derivative at one given time and one location in space, but it is the change of V_{MAX} at two different times and two different locations due to the change of the RMW with time. In their reply to this comment, Stern et al. (2017, hereafter S17) interpret $(dV/dt)_{\mathrm{RMW}}$, however, as the total derivative at one specific time and point in space; that is,

$$\left(\frac{dV}{dt}\right)_{t=\tau, r=\mathrm{RMW}} = \left.\frac{\partial V}{\partial t}\right|_{t=\tau, r=\mathrm{RMW}} + \left.\frac{\partial V}{\partial r}\frac{dR}{dt}\right|_{t=\tau, r=\mathrm{RMW}} \qquad (6b)$$

[see Eq. (4) in S17] and argue that $(dV/dt)_{\mathrm{RMW}}$ is then the same as $(\partial V/\partial t)_{\mathrm{RMW}}$, because $\partial V/\partial r = 0$ at the RMW. Based on this interpretation, S17 claim that Eq. (5) cannot be derived from the directional derivative because of the presence of the zero denominator. S17's argument is invalid because $(dV/dt)_{\mathrm{RMW}}$ in W82 and $(dV/dt)_{t=\tau,\mathrm{RMW}}$ in Eq. (6b) represent two different concepts: the former, given in Eq. (6a), is used to estimate the temporal changes of V following the contracting RMW [i.e., $(V_B - V_A)/\Delta t$ in Fig. 1], whereas the latter, after setting $\partial V/\partial r = 0$ in Eq. (6b) by S17, is used to calculate the total temporal changes of V at one fixed RMW and time [i.e., $(V_D - V_A)/\Delta t$ in Fig. 1]. The two are only identical when the RMW or V_{MAX} does not change with time. From the practical standpoint, W82's calculation of $(dV/dt)_{\mathrm{RMW}}$ can be easily estimated from flight data by simply subtracting the V_{MAX} values at two different times and dividing it by the time interval, which has also been demonstrated in Fig. 1. Thus, one should not simply replace $(dV/dt)_{\mathrm{RMW}}$ estimated over a finite time interval in W82's original equation by $(\partial V/\partial t)_{\mathrm{RMW}}$ as argued in S17.

Even if we could accept that the interchange between the above two derivatives at the RMW, the second-order derivative given by Eq. (3) using the left, the right, and the centered finite difference at the RMW, will result in some ambiguity that any attempt to match the finite-difference form of Eq. (3) with W82's original equation would be misleading. Specifically, S17's use of one-side derivative to relate Eq. (3) to Eq. (5) herein is subjective and arbitrary, because the RMW contraction rate could differ, depending on how Eq. (3) is calculated. For instance, if a more accurate finite approximation, such as a centered finite difference, is used to calculate the RMW tendency in Eq. (3), that is,

$$\frac{dR}{dt} = -\frac{\left(\frac{\partial V}{\partial t}\right)_{\mathrm{RMW}+\Delta R} - \left(\frac{\partial V}{\partial t}\right)_{\mathrm{RMW}-\Delta R}}{\left(\frac{\partial V}{\partial r}\right)_{\mathrm{RMW}+\Delta R} - \left(\frac{\partial V}{\partial r}\right)_{\mathrm{RMW}-\Delta R}},$$

then there is no way one can get rid of the term $(\partial V/\partial r)_{\mathrm{RMW}+\Delta R}$ in the denominator when relating Eq. (3) to Eq. (5). Thus, the similarity between Eqs. (3) and (5) does not apply for this second-order accuracy approximation. Similarly, calculating $\partial V/\partial r$ first and then $\partial(\partial V/\partial r)/\partial t$ versus calculating $\partial V/\partial t$ first and then $\partial(\partial V/\partial t)/\partial r$ at the RMW in Eq. (3) may also lead to different results. From this perspective, choosing one particular finite approximation to prove that two mathematical expressions are the same as presented in S17 is obviously an invalid mathematical argument. In summary, we have shown from the above several angles that unlike S15 and S17 claim, S15's RMW tendency equation is not similar to W82's [cf. Eqs. (3) and (5) herein].

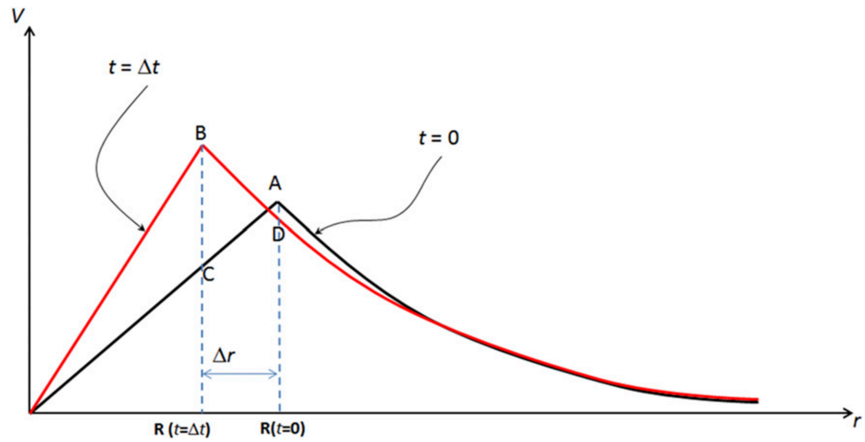

FIG. 1. Schematic of the RMW contraction model presented in W82. Black (red) line denotes the radial profile of the tangential wind at a given level at time $t = 0$ ($t = \Delta t$).

3. A dynamical interpretation of the RMW contraction

In early analyses of idealized simulations using an axisymmetric model and ensemble real-data simulations of Hurricane Katrina (2005) with the Weather Research and Forecast (WRF) Model, K12 noted a cessation of the RMW contraction (i.e., a steady state in the RMW) during TC rapid intensification, regardless of model configurations or initialization. This steady state in the RMW contraction turns out to be common during the rapidly intensifying stages of TCs from the best track data (Qin et al. 2016) and many previous modeling studies of TC development (e.g., Chen et al. 2011; K12; Wang and Wang 2014) but has not been well understood. To examine this phenomenon, K12 used a simple Rankine-like vortex to derive the physical mechanisms underlying the contraction of the RMW. Despite its simplicity, K12's kinematic model suffices to arrive at a conclusion that the RMW contraction is governed by two dominantly opposing processes: one is related to the inward advection of the absolute angular momentum (AAM) by the radial inflow, while the other is attributed to the frictional forcing in the planetary boundary layer (PBL) that tends to oppose the RMW contraction. The contraction will stop if the frictional forcing, which is proportional to the square of V_{MAX}, can be balanced by the inward advection by the radial inflow.

An interesting feature of K12's model is that the condition for the RMW to stop contraction still allows the vortex to intensify, provided that the inward advection of the absolute angular momentum by the radial inflow at the RMW can be maintained. This simple model thus captures the cessation or interruption of RMW contraction during TC intensification as found from previous modeling studies (Chen et al. 2011; K12; Wang and Wang 2014). The role of frictional dissipation in preventing the RMW from collapsing is confirmed in more detailed analyses by Castaño et al. (2014) and N. Qin et al. (2017, unpublished manuscript). Although K12's model contains an inherent weakness in assuming a given profile for the radial wind $u(r, t)$ that is governed by the radial momentum equation, it could at least reveal an important role of the PBL friction in the inner-core region in suppressing the RMW contraction.

In their discussion of the RMW contraction, S15 commented that there is "a mathematical error," that is, missing a term "VdR/dt," in K12's derivation (footnote 10 in S15, p. 1300), which invalidates the main result related to the impacts of the frictional forcing. Furthermore, S15 argued that the contraction of the RMW should depend only on the "the tangential wind near the RMW and its radial and time derivatives" (S15, p. 1300) but not on the friction nor the radial wind, because S15 argued that the strong radial inflow would otherwise imply a collapse of the RMW contraction. We found that S15's comments on the mathematical error in K12 and especially their related discussion about the role of friction in the RMW contraction are invalid.

To facilitate our discussion, let us summarize below the main steps by which K12 obtained his Eq. (5). Basically, K12 started with a barotropic Rankine-like vortex structure given by

$$v(r,t) = \begin{cases} \Omega(t) r & \forall r < R \\ \dfrac{\partial v(r,t)}{\partial r} = 0 & \text{at} \quad r = R \end{cases} \tag{7}$$

and proposed two questions: (i) How will the Rankine-like vortex as given by Eq. (7) contract with time, and (ii) could it be used to explain the cessation of the RMW contraction during rapid intensification as often found from full-physics models and observations?

To answer the above questions, K12 used the axisymmetric tangential momentum equation as follows:

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial r} - \frac{uv}{r} - w\frac{\partial v}{\partial z} - fu - \frac{C_D}{H}v\sqrt{(u^2+v^2)}, \qquad (8)$$

where the last term on the rhs denotes frictional dissipation in the PBL. Substituting the radial profile Eq. (7) into Eq. (8), and noting that (i) $\partial v/\partial z = 0$ since $v(r, t)$ does not depend on z and (ii) the partial derivative with respect to t will not act on the coordinate variable r, we obtain

$$\frac{d\Omega}{dt}r = -u\frac{\partial v}{\partial r} - (\Omega + f)u - \frac{C_D}{H}v\sqrt{(u^2+v^2)} \quad \forall r \le R, \qquad (9a)$$

where

$$\begin{cases} \dfrac{\partial v(r,t)}{\partial r} = \Omega(\mathrm{t}) & \forall r < R \\ \dfrac{\partial v(r,t)}{\partial r} = 0 & \text{for} \quad r = R \end{cases} \qquad (9b)$$

[see appendix 1 in K12 for constructing a piecewise smooth profile satisfying the condition in Eq. (9b)[2]]. Note that because Ω is assumed to be a strict function of time for the Rankine-like vortex in all K12's derivations, that is, $\Omega = \Omega(t)$, it is justifiable to replace $\partial\Omega/\partial t$ by $d\Omega/dt$ in Eq. (9a). Since Eq. (9a) is valid for $\forall r \le R$, it has to be valid at $r = R$. Evaluating Eq. (9a) at $r = R$, and noting the condition Eq. (9b), we have

$$\frac{d\Omega^*}{dt}R = -(\Omega^* + f)U - \frac{C_D}{H}V\sqrt{U^2+V^2} \quad \text{at} \quad r = R, \qquad (10)$$

where $U(t) = u(r, t)|_{r=R}$ and the notation $\Omega^*(t)$ is used to emphasize that the angular velocity is exactly defined at $r = R$ as follows:

$$V(t) = v(r, t)|_{r=R} = \Omega^*(t)R. \qquad (11)$$

In practice, $\Omega^*(t)$ is slightly different from $\Omega(t)$ defined in Eq. (9b) when a piecewise smooth radial profile of $V(t)$ given by Eq. (9b) is used. This small difference between $\Omega^*(t)$ and $\Omega(t)$ does not, however, change K12's RMW contraction equation, because all of K12's derivations assume that $\Omega^*(t) = \Omega(t)$, which is well justified for an actual wind profile (see the appendix). We note in the above discussion that, at the RMW, Eq. (11) is used to estimate V_{MAX}, but Eq. (9b) must be used to estimate the radial derivative of $v(r, t)$. These two equations are not interchangeable. Therefore, one should not use Eq. (11) to estimate the radial derivative of $v(r, t)$ at the RMW.

For the sake of consistency herein, all uppercase letters denote the variables evaluated at the RMW to distinguish from the lowercase letters representing Eulerian field variables. Because $U(t) < 0$, Eq. (10) can be rewritten as

$$\frac{d\Omega^*}{dt}R = (\Omega^* + f)|U| - \frac{C_D}{H}V\sqrt{U^2+V^2} \quad \text{at} \quad r = R. \qquad (12)$$

This is essentially Eq. (5) in K12, except that here, we use Ω^* instead of Ω when applying Eq. (9a) at $r = R$ for the clarity in this comment. Basically, Eq. (12) describes how a Rankine vortex given by Eq. (7) would evolve with time. It states that if a vortex with a tangential wind distribution given by Eq. (7) follows the governing Eq. (8), then at $r = R$, the evolution of the angular velocity $\Omega^*(t)$ must be governed by Eq. (12). We note that Eq. (12) is not closed because $U(t)$ is not known, which is ultimately linked to the radial momentum equation, the equation of state, and the thermodynamic equation.

To estimate the RMW contraction rate, K12 assumed that the profile in Eq. (7) will be maintained at all times during the contraction. This implies that the Rankine relationship $V(t) = \Omega^*(t)R(t)$ at the RMW must be valid during contraction so that upon taking a time derivative of this Rankine relationship along the contracting direction given by $R(t)$, we have

$$\frac{dV}{dt} = \frac{d\Omega^*}{dt}R + \Omega^*\frac{dR}{dt}, \qquad (13)$$

which can be rearranged as

$$\frac{d\Omega^*}{dt}R = \frac{dV}{dt} - \Omega^*\frac{dR}{dt}. \qquad (14)$$

We note again that Eq. (13) does not imply that the RMW has to follow any momentum or material surface along a given streamline. It merely states that if the Rankine-like profile $v(r, t) = \Omega(t)r$ is maintained during the entire TC development, then the value of the field variable $v(r, t)$ at the point $r = R(t)$, that is, $V(t) = v[R(t), t]$, must always satisfy $V(t) = \Omega^*(t)R(t)$ and so Eq. (13) is ensured. Plugging Eq. (14) into Eq. (12), we arrive at

$$\Omega^*\frac{dR}{dt} = \frac{dV}{dt} - (\Omega^* + f)|U| + \frac{C_D}{H}V\sqrt{U^2+V^2} \quad \text{at} \quad r = R, \qquad (15)$$

[2] An example of a smooth wind profile $v(r, t)$ that ensures condition Eq. (9b) is $v(r, t) = \Omega(t)re^{-(r/R)^{20}} + \beta[1 - e^{-(r/R)^{20}}]/r^{\alpha}$, where β and α are proportional coefficients. This smooth profile, plotted in Fig. A1, has a typical Rankine property that (i) $v(r, t) \approx \Omega(t)r$ for $r < R$, (ii) $v(r, t) \approx \beta/r^{\alpha}$ for $r > R$, and (iii) $\partial v(r, t)/\partial r = 0$ at $r \approx R$.

which is Eq. (6) in K12. If one notes that $\Omega^* \gg f$, $V \gg U$, then a slight rearrangement of Eq. (15) gives

$$\frac{dR}{dt} = -|U| + \frac{C_D}{H}VR + \frac{R}{V}\frac{dV}{dt}, \tag{16}$$

which is Eq. (7) in K12 with no difference.

An immediate consequence of Eq. (16) is that the RMW contraction rate will be faster for larger drag coefficient C_D. This result can be readily confirmed in any model framework by simply varying the coefficient C_D and seeing how the rate of the RMW contraction looks (see Fig. 4 and related discussion in K12). Figure 2 reproduces the results from this C_D-sensitivity experiment in K12, using the axisymmetric model developed by Rotunno and Emanuel (1987). As C_D increases from 0.8×10^{-3} to 1.2×10^{-3}, we notice that the RMW contraction rate (i.e., the slope of the RMW time series) increases during the rapid intensification period. This confirms the role of the PBL friction in governing the RMW contraction process as suggested by Eq. (16).

Unlike S15 and S17 misinterpret, the RMW contraction rate [i.e., Eq. (16)] by no means implies that the RMW is a surface or momentum surface advected by the radial inflow. As discussed in K12, the first term on the rhs of Eq. (16) represents the impacts of the inward advection of the AAM by the radial inflow. During the TC intensifying stage, this inward AAM advection leads to the spinup of the tangential wind in the inner-core region, thus continuously producing a new RMW inward that pictorially gives one an impression that the RMW is advected inward by the radial inflow. On the other hand, friction [i.e., the second term on the rhs of Eq. (16)] tends to reduce the spinup of the tangential wind, thus preventing the formation of any newly formed RMW. As a result of the larger AAM advection during the TC intensification, the RMW will be contracted until friction can balance the AAM advection. From this physical perspective, it is entirely possible that the RMW can expand even within the inflow regime if the friction becomes more dominant, for example, during landfall [i.e., the second term is larger than the first term in Eq. (16)].

Because of their misinterpretation of our RMW contraction equation and their dismissing the role of friction, S17 mislead the readers by arguing that our equation must always produce the RMW contraction in the inflow regime. S17's argument about the role of friction is physically irrational, because modeling analyses show that the RMW is located at the radius where the radial inflow is strong, that is, $U < 0$. Therefore, the existence of frictional force is a must to prevent in the RMW contraction from shifting inwards, as indicated by Eq. (16). A vortex structure with a strong inflow at the RMW but no frictional force as S15 argued is not physically realizable, because the PBL frictional convergence is part of TC intensification. Without friction, a TC cannot even exist let alone allow for the existence of a radial inflow at the RMW. The importance of friction in governing the RMW contraction was observed in an earlier modeling study by Yau et al. (2004), who showed that the simulated RMW associated with Hurricane Andrew (1992) increases as the surface friction is reduced. It is therefore perplexing to see S17's strong claim that the friction is not directly related to the RMW contraction as articulated in their discussion section.

Along with their unjustified dismissing of the role of friction in the RMW contraction, S15 was also confused about the partial time derivative $\partial v/\partial t$ versus the total derivative dv/dt in going from Eq. (8) to Eq. (12) in their earlier discussion (see footnote 10 in S15, p. 1300). Note that the partial time derivative $\partial v/\partial t$ in Eq. (8) will not touch upon the coordinate variable r. K12 simply plugged a functional form $v(r, t) = \Omega(t)r$ into the partial derivative $\partial v/\partial t$ to get a term $(d\Omega/dt)r$ as seen in Eq. (9a) and evaluated the resulting Eq. (9a) at the point $r = R$ to arrive at Eqs. (10) and (12). Therefore, one should not expect to have "the term $\Omega dR/dt$" in Eqs. (9a) and (9b) as S15 (p. 1300) comment.

Unfortunately, S17 provide no response to our above remark in their reply to this comment, which is one of the major purposes of this comment and reply. Instead, S17 introduce a new assumption that the Rankine parameter Ω should be now a function of both time and radius and then use this new assumption to criticize K12's derivation. It should be pointed out that K12 has never assumed that $\Omega = \Omega(r, t)$ as seen from the above derivations of Eqs. (7)–(16). Rather, $\Omega(t)$ is assumed to be only a function of time in all K12's derivations such that $\Omega^*(t) = \Omega(t)$, and so $d\Omega/dt = \partial\Omega/\partial t$ (see the appendix for the justification of this assumption). Apparently, if $\Omega(t)$ is a function of time only, then the extra term in S17's Eq. (23) will vanish, and S17's Eq. (23) is identical to K12's Eq. (6) or Eq. (15) herein. Of course, one can always question the validity of any assumption, but it is not a mathematically valid argument to use a different assumption to claim that another derivation has "a mathematical error" simply because different assumptions lead to different equations. Their inconsistent comments of K12's missing a term VdR/dt in S15 (see footnote 10 in S15, p. 1300) and K12's missing a term $(dR/dt)(\partial\Omega/\partial t)$ in S17 [see S17, their Eq. (23)] indicates that the coauthors of S15 and S17 are unwilling to accept their mistake in commenting on K12's

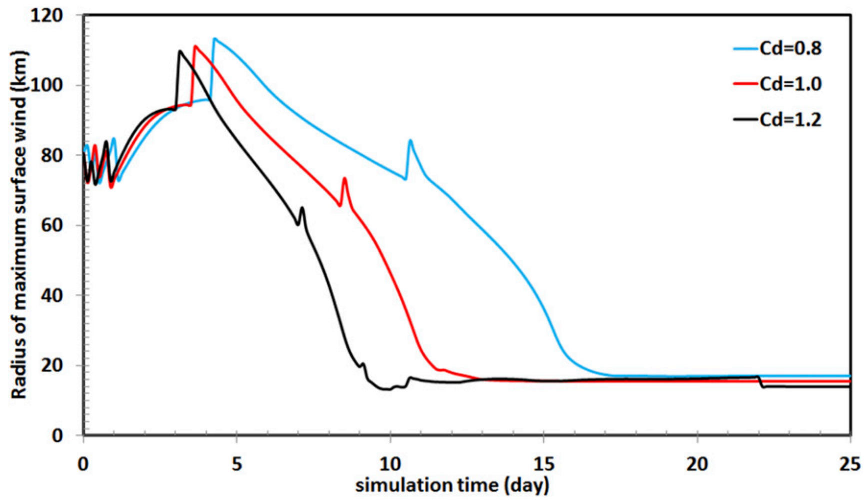

FIG. 2. Time series of the RMW (km) from Rotunno and Emanuel's (1987) axisymmetric hurricane model obtained from a 25-day simulation, with the surface drag $C_D = 0.8 \times 10^{-3}$ (black), 1.0×10^{-3} (red), and 1.2×10^{-3} (cyan). Reproduced from K12.

derivation. This can also be seen from their reply to our comments on the application of their geometrical model for the RMW contraction and their arguing for the similarity of their geometrical model to W82's equation of the RMW tendency.

It should be pointed out that the coauthors of S15 and S17 appear to understand clearly the well-known condition of $\partial v/\partial r = 0$ at the RMW, since they have used it (i) to derive their geometric model and (ii) to claim the similarity of their RMW tendency equation to W82's. Thus, their new assumption of $\Omega(r, t)$ as functions of both r and t when taking a radial derivative of $v(r, t)$ at the RMW is irrational and unjustified. As indicated clearly in Eq. (9b), we have $v(r, t) = \Omega(t)r$ for $r < R$, but we must have exactly $\partial v/\partial r = 0$ when taking the radial derivative of v at $r = R$. The smooth profile in our footnote 2 is an example that could ensure the condition Eq. (9b), so there is no conflict in our wind profile as S17 argue; that is, our conditions $\partial v/\partial r|_{r=R} = 0$ and $V(t) = \Omega^*(t)R(t)$ are not mutually exclusive. Despite this clear emphasis, that S17 still assume the profile $v(r, t) = \Omega(r, t)r$ at the RMW to criticize our derivation indicates their unwillingness to accept their confusion in S15. So we state that S17's argument based on the dependence of $\Omega(r, t)$ on radius is irrelevant to our derivations, because we only work with $\Omega(t)$ rather than with the full profile $\Omega(r, t)$ as S17 claim.

One may notice that S17's use of a different wind profile from that used in K12 to attack the validity of K12's derivation implies S17's indirect acknowledgment of misinterpretation in S15 and that there is no mathematical error in K12's RMW contraction equation. Moreover, S17 (section 5) use the diagnostically obtained results for the mature stage of a simulated TC to attack the results of K12 and those presented herein, which are derived from the prognostic tangential momentum equation for the intensifying stage of a TC. Apparently, they fail to realize that by doing so, they are comparing apples and oranges, because these results are obtained from two different types of equations for two different stages of TC development. Moreover, they also fail to understand that the radial gradient of the time tendency of V, that is, $(\partial/\partial r)(\partial V/\partial t)$, and the curvature of the radial profile of V (i.e., $\partial^2 V/\partial r^2$) in their geometrical model [i.e., Eq. (3) herein] must involve dynamical processes that are dictated by the prognostic tangential momentum equation [e.g., Eq. (8)]. Despite their misinterpretations and inconsistencies between S15 and S17, one notices S17's excessive uses of unusual expressions to negate K12's derivations and our comments, which are either not substantiated or represent their misinterpretations or misunderstandings.

Although K12 did not discuss the evolution and structures of a TC vortex in the outer region $r > R$, a quick inspection of a full Rankine profile will show that it is not likely to have a situation in which the inflow can lead to expansion of the RMW by allowing for the tangential wind outside of the RMW to spin up faster than that in the inner-core region. Indeed, consider the radial wind profile Eq. (7) with a further expansion for the outer-core region $r > R$ as follows:

$$v(r,t)=\begin{cases}\Omega(t)r & \forall r<R\\ \dfrac{\Omega(t)R^2}{r} & \forall r>R\\ \dfrac{\partial v}{\partial r}=0 & \text{at}\quad r=R.\end{cases}\tag{17}$$

Plugging Eq. (17) into Eq. (8) gives

$$\begin{cases}\dfrac{\partial v}{\partial t}=-(2\Omega+f)u-\dfrac{C_D}{H}v\sqrt{u^2+v^2} & \forall r<R\\ \quad\dfrac{\partial v}{\partial t}=-fu-\dfrac{C_D}{H}v\sqrt{u^2+v^2} & \forall r>R.\end{cases}\tag{18}$$

It is immediately clear from the rhs of Eq. (18) that the positive contribution to the spinup of the inner-core winds $-(2\Omega+f)u$ is much larger than the spinup of the outer-region winds, which is proportional to $-fu$. One should not therefore expect a situation in which the friction alone could account for the weakening of tangential winds in the inner-core region as discussed in S15 (p. 1301), because the advective term $-(2\Omega+f)u$ will dominate during the TC rapid intensification period. In fact, detailed derivations in Kieu and Zhang (2009) show that such a drastic difference in the absolute angular momentum advection between the inner-core and the outer-core regions explains the much faster spinup of the tangential flow in the former than that in the latter region. As a result, a vortex with a deceleration or a much slower spinup of the tangential wind in the inner-core region relative to the spinup in the outer-core region should not occur during the rapid intensification of TCs, at least, for the Rankine-like profiles Eqs. (7) and (17).

4. Summary and conclusions

In this comment, we have reexamined the issue of RMW contraction during TC development as studied by K12 and S15. We have shown that (i) S15 mistakenly commented on a missing term VdR/dt in K12's derivation of an equation for the RMW contracting rate and (ii) S15's geometrical model provides little dynamical understanding of the RMW contraction because it does not involve any equation in the primitive equations system. Although S15's statements such as "contraction begins and subsequently accelerates as a result of an increase in the radial gradient of the wind tendency" (S15, section 5a) or "acceleration of contraction is due to an increasing radial gradient of the V tendency" (S15, p. 1290) are not wrong, they bear little physical implication as the key question is, Why could "an increase in the radial gradient of the wind tendency" occur during TC development?

Along with the issue about the interpretation of the RMW contraction in their geometrical model, we also showed that S15 made a mathematical error in their argument that incorrectly led them to a conclusion that S15's equation of the RMW contraction is equivalent to W82's equation of the RMW contraction. From the mathematical error in relating their RMW tendency equation to W82's and their erroneous comment of a missing term VdR/dt in K12's derivation, we found that S15 were confused in both cases about the partial time derivative $\partial v/\partial t$ versus the total derivative dv/dt.

Finally, we have pointed out that in their reply to this comment, S17 provide no response to our comment on their mistaken statement in S15 that K12's derivations miss a term VdR/dt. Instead, S17 use a radial profile of the tangential wind that differs from that in K12 to claim that K12's derivations miss a term, $(dR/dt)(\partial\Omega/\partial r)R$. In this comment, we have shown that (i) S17's use of a different radial profile of the tangential wind from that used in K12 to attack K12's derivations is unjustified and (ii) S17's claim of missing the term $(dR/dt)(\partial\Omega/\partial r)R$ in K12 is rooted in their using $\partial v(r,t)/\partial r\neq 0$ at the RMW to derive K12's RMW tendency equation. We could see that the coauthors of S17 understand well the condition of $\partial v/\partial r = 0$ at the RMW, since they have used it to derive their geometric model and to claim the similarity of their RMW tendency equation to W82's. However, S17's use of a different wind profile $v(r,t)=\Omega(r,t)r$ from that in K12 and their lacking response to our comment indicate their unwillingness to accept their mistaken comments in S15. Based on the above analyses, we conclude that K12's derivations for the RMW contraction equation contain no mathematical error and that the associated reply and discussion in S17 are irrelevant to our comments on S15's misinterpretation.

Acknowledgments. This work was supported by ONR Grant N000141410143.

APPENDIX

Approximation for the Inner-Core Angular Velocity Ω at the RMW

There is an implicit assumption in K12 that the angular velocity estimated at the RMW, which is denoted by $\Omega^*(t)$ in Eq. (10) herein, is the same as the angular velocity $\Omega(t)$ in the inner-core region of a hurricane-like vortex. This assumption lies at the root of the criticism that S17 introduce in their reply to this comment, as S17 argue that this difference is significant at the RMW such that $\partial\Omega/\partial r|_{r=R}\neq 0$, and so it cannot be neglected. In this appendix, we will show that under the

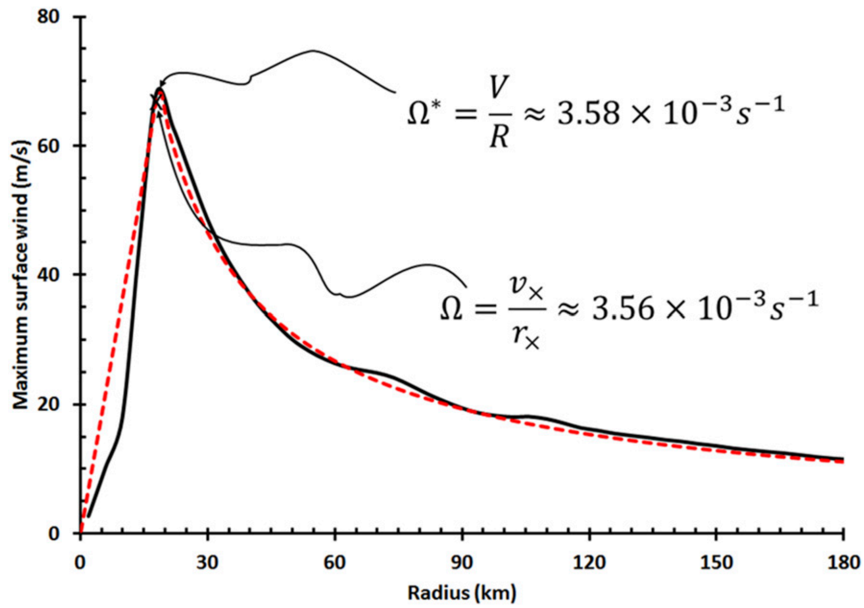

FIG. A1. The radial profile of the tangential wind (solid black; m s^{-1}) at 10-m height that is obtained from Rotunno and Emanuel's (1987) axisymmetric hurricane model after 72 h into the integration. The cross denotes the point near the RMW with an angular velocity of $\Omega \sim 3.56 \times 10^{-3}$ s^{-1}, along with an angular velocity of $\Omega^* \sim 3.58 \times 10^{-3}$ s^{-1} that is evaluated exactly at the RMW location. The dashed red curve denotes the analytical profile $v(r, t) = \Omega(t) r e^{-(r/R)^{20}} + \beta[1 - e^{-(r/R)^{20}}]/r^\alpha$ with $\Omega(t) \sim 3.58 \times 10^{-3}$ s^{-1}, $R = 19$ km, $\alpha = 0.8$, and $\beta = \Omega R^{\alpha+1}$.

piecewise smooth profile given by condition Eq. (9b) in the main text, $\Omega^*(t) \approx \Omega(t)$ such that $d\Omega^*/dt \approx d\Omega/dt$ as assumed in K12.

Indeed, we first apply the Taylor expansion for the tangential wind $v(r, t)$ at $r = R$ as follows:

$$v(R - \delta r, t) \approx v(R, t) - \left.\frac{\partial v}{\partial r}\right|_{r=R} \delta r + O(\delta r^2) \cdots, \quad \text{(A1)}$$

where δr is a small increment around R and $O(\delta r^2)$ denotes higher-order terms. Because of the piecewise smooth condition Eq. (9b), and recall that $V(t) = v(R, t)$, we rearrange Eq. (A1) to obtain

$$V(t) \approx v(R - \delta r) + O(\delta r^2) = \Omega(t) \times (R - \delta r) + O(\delta r^2). \quad \text{(A2)}$$

With the definition of $\Omega^*(t)$ at the RMW, dividing R at both sides of Eq. (A2) gives

$$\Omega^*(t) \approx \Omega(t)\left(1 - \frac{\delta r}{R}\right) + O(\delta r^2). \quad \text{(A3)}$$

Apparently, in the limit of $\delta r \to 0$, $\Omega^*(t)$ is as close to $\Omega(t)$ as expected, which underlines K12's assumption of $\Omega(t)$ as a strict function of time $\forall r \le R$ in deriving Eq. (10) herein. One could in principle introduce a new assumption to take into account the variation of Ω with radius at the RMW. However, the physical implications of the frictional force and the AAM advection in the RMW contraction as described by our Eq. (16) continue to be valid from the dynamical perspective.

To directly verify the approximation of $\Omega^* \approx \Omega$ from the model output, Fig. A1 displays an example of the radial profile of the tangential wind at the model lowest level as obtained from Rotunno and Emanuel's (1987) axisymmetric model after 72 h into the model integration. A straightforward calculation of Ω^* at the RMW location gives $\Omega^* \sim 3.58 \times 10^{-3}$ s^{-1}, and similar calculation for Ω at a point nearby gives $\Omega \sim 3.56 \times 10^{-3}$ s^{-1}. Apparently, Ω^* and Ω are very close as expected, thus justifying the assumption of $\Omega^* \approx \Omega$ discussed below Eq. (10).

As a final check of how well the modified Rankine profile with a constant solid-body rotation Ω assumed in K12 can be applied to the actual TC inner-core region, the analytical profile provided in footnote 2 is also plotted in Fig. A1, assuming that $\Omega = 3.58 \times 10^{-3}$ s^{-1}, $R = 19$ km, $\alpha = 0.8$, and $\beta = \Omega R^{\alpha+1}$. Note again that this smooth profile has all typical Rankine properties including (i) $v(r, t) \approx \Omega(t) r$ for $r < R$, (ii) $v(r, t) \approx \beta/r^\alpha$ for $r > R$, and (iii) $\partial v(r, t)/\partial r = 0$ at $r \approx R$ as assumed in Eq. (9b). Except for some distortion of the wind distribution near the vortex center, one notices that the analytical profile describes well the wind structure in the TC eye as a solid-body rotation with a constant angular velocity inside the inner-core region. Such an approximated linear structure of the tangential wind in the TC inner core is well maintained after the model storm reaches $30\,m\,s^{-1}$, at least in the axisymmetric model framework. Of course, one could refine further this wind structure to better capture the inner-core tangential wind. However, the Rankine profile with the angular velocity as a strict function of time is practically acceptable, at least to the zero order as assumed in K12's model [see also Holland et al. (2010) for a review of modified Rankine profiles for TC wind structure].

REFERENCES

Castaño, D., M. C. Navarro, and H. Herrero, 2014: Thermoconvective vortices in a cylindrical annulus with varying inner radius. *Chaos*, **24**, 043116, https://doi.org/10.1063/1.4898732.

Chen, H., D.-L. Zhang, J. Carton, and R. Atlas, 2011: On the rapid intensification of Hurricane Wilma (2005). Part I: Model prediction and structural changes. *Wea. Forecasting*, **26**, 885–901, https://doi.org/10.1175/WAF-D-11-00001.1.

Corbosiero, K. L., J. Molinari, and M. L. Black, 2005: The structure and evolution of Hurricane Elena (1985). Part I: Symmetric intensification. *Mon. Wea. Rev.*, **133**, 2905–2921, https://doi.org/10.1175/MWR3010.1.

Holland, G. J., J. I. Belanger, and A. Fritz, 2010: A revised model for radial profiles of hurricane winds. *Mon. Wea. Rev.*, **138**, 4393–4401, https://doi.org/10.1175/2010MWR3317.1.

Kieu, C. Q., 2012: An investigation into the contraction of the hurricane radius of maximum wind. *Meteor. Atmos. Phys.*, **115**, 47–56, https://doi.org/10.1007/s00703-011-0171-7.

——, and D.-L. Zhang, 2009: An analytical model for the rapid intensification of tropical cyclones. *Quart. J. Roy. Meteor. Soc.*, **135**, 1336–1349, https://doi.org/10.1002/qj.433.

Petterssen, S., 1956: *Motion and Motion Systems.* Vol. I, *Weather Analysis and Forecasting*, McGraw-Hill, 428 pp.

Qin, N., D.-L. Zhang, and Y. Li, 2016: A statistical analysis of steady eyewall sizes associated with rapidly intensifying hurricanes. *Wea. Forecasting*, **31**, 737–742, https://doi.org/10.1175/WAF-D-16-0016.1.

Rotunno, R., and K. A. Emanuel, 1987: An air–sea interaction theory for tropical cyclones. Part II: Evolutionary study using a nonhydrostatic axisymmetric numerical model. *J. Atmos. Sci.*, **44**, 542–561, https://doi.org/10.1175/1520-0469(1987)044<0542:AAITFT>2.0.CO;2.

Schutz, B. F., 1980: *Geometrical Methods of Mathematical Physics.* 1st ed. Cambridge University Press, 264 pp.

Stern, D. P., J. L. Vigh, D. S. Nolan, and F. Zhang, 2015: Revisiting the relationship between eyewall contraction and intensification. *J. Atmos. Sci.*, **72**, 1283–1306, https://doi.org/10.1175/JAS-D-14-0261.1.

——, ——, ——, and ——, 2017: Reply to "Comments on 'Revisiting the relationship between eyewall contraction and intensification.'" *J. Atmos. Sci.*, https://doi.org/10.1175/JAS-D-17-0120.1, 4275–4286.

Wang, H., and Y. Wang, 2014: A numerical study of Typhoon Megi (2010). Part I: Rapid intensification. *Mon. Wea. Rev.*, **142**, 29–48, https://doi.org/10.1175/MWR-D-13-00070.1.

Willoughby, H. E., J. A. Clos, and M. G. Shoreibah, 1982: Concentric eye walls, secondary wind maxima, and the evolution of the hurricane vortex. *J. Atmos. Sci.*, **39**, 395–411, https://doi.org/10.1175/1520-0469(1982)039<0395:CEWSWM>2.0.CO;2.

Xu, J., and Y. Wang, 2010: Sensitivity of the simulated tropical cyclone inner-core size to the initial vortex size. *Mon. Wea. Rev.*, **138**, 4135–4157, https://doi.org/10.1175/2010MWR3335.1.

Yau, M. K., Y. Liu, D.-L. Zhang, and Y. Chen, 2004: A multiscale numerical study of Hurricane Andrew (1992). Part VI: Small-scale inner-core structures and wind streaks. *Mon. Wea. Rev.*, **132**, 1410–1433, https://doi.org/10.1175/1520-0493(2004)132<1410:AMNSOH>2.0.CO;2.