A Field Deformation Approach to Spatio-Temporal Forecast Verification of Gridded Sets

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Spatial Forecast Verification Methods

Inter-Comparison Project (ICP)

 Filtering
- neighborhood
- scale-separation

 Filtering
- feature-based
- field deformation

http://www.ral.ucar.edu/projects/icp
Displacement Methods: Field deformation

**Goal:** Inform about how well the forecast captures spatial extent/patterns.

**Examples:**

**Binary Image Metrics** (Venugopal *et al.*, 2005; G. 2011; Schwedler and Baldwin, 2011; Zhu *et al.*, Submitted)

**Optical Flow** (e.g., Keil and Craig, 2008, 2009)

**Image Warping** (e.g., Alexander *et al.*, 1998; G., Lindström and Lindgren, 2010)

**Distortion representation** (e.g., Hoffman *et al.*, 1995)
Displacement Methods: Field Deformation

Field Deformation Methods: Image Warping

\[ O(x, y) = F(W_x(x, y), W_y(x, y)) + \varepsilon \]
Displacement Methods: Field Deformation

Field Deformation Methods: Image Warping

\[ O(x, y) = F(W_x(x, y), W_y(x, y)) + \varepsilon \]

- \( W \) is a warping function that acts on both coordinates \( x \) and \( y \) of an image, and is applied to both coordinates;

- Many choices for \( W \), e.g.,
  - polynomials (e.g., Alexander et al., 1999; Dickinson and Brown, 1996)
  - B-splines (e.g., Engel in prep?)
  - Thin-plate splines (e.g., G., Lindström and Lindgren, 2010)

- Find optimal warp by optimizing a likelihood function.
Displacement Methods: Field Deformation

Field Deformation Methods: Image Warping

TPS warp function is a linear function in the $1$-energy control points. That is,

$$W(s, p^O, p^F) = B(s, p^O)p^F$$

where $B$ is a matrix of radial basis functions that is \emph{pre-calculated}. 
Displacement Methods: Field Deformation

Field Deformation Methods: Image Warping
Optimize (log) likelihood:
\[
\ell(p^F|O, F, p^O) = \log p(O|F, p^F, p^O) + \log p(p^F|p^O) + \log p(\vartheta)
\]

- Intensity component
- Location/spatial placement component
- Possibly hyper-parameters
Displacement Methods: Field Deformation

Field Deformation Methods: Image Warping

For the TPS Warp, the following optimization function can be used (assumes Gaussian errors, and a Markov Random Field Model for the control point differences).

\[
Q(p^F) = \frac{1}{2\sigma^2} \sum (O(s) - F(W(s)))^2 + \frac{1}{2\sigma^2_\Delta} \left[ (p_x^F - p_x^O)^T (I - C)(p_x^F - p_x^O) + (p_y^F - p_y^O)^T (I - C)(p_y^F - p_y^O) \right]
\]

e.g., Åberg et al., *Environmetrics*, 16(8):833–848, 2005.
ICP Test Cases

Forcast
Observation
Deformed forcast

MSE 671.32
MSE 0.27
Warp $-3.39 \times 10^{-3}$
x: 33.3    y: 0.1
sx: 0.252    sy: 1.029

Geometric 3; 125 grid points too far east and larger spatial coverage
≈ 100 grid points west
Squeezes horizontally.
\[
\text{MSE(before)} = 17,508 \quad \frac{17,508 - 9,316}{17,508} \approx 47\% 
\]
Space-Time Image Warp

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Space-Time Image Warp

Can timing errors be distinguished from spatial displacement errors?

Extension of 2-d spatial warping to space-time

Equations about the same, but with the added dimension. Tri-harmonic basis functions instead of 2-d TPS radial basis functions.
Space-Time Image Warp

Can timing errors be distinguished from spatial displacement errors?

Extension of 2-d spatial warping to space-time

\[
Q(p^F) = \frac{1}{2\sigma^2_{\varepsilon}} \sum (O(s) - F(W(s)))^2 + \\
\frac{1}{2\sigma^2_{\Delta}} \left[ (p^F_x - p^O_x)^T (I - C)(p^F_x - p^O_x) + (p^F_y - p^O_y)^T (I - C)(p^F_y - p^O_y) \right] + \\
\frac{1}{\sigma^2_t} \left[ (p^F_t - p^O_t)^T (I - C)(p^F_t - p^O_t) \right]
\]
Space-Time Image Warp

Example

Reduction in RMSE is over 50% after applying space-time warp. Most errors were spatial only.
Final Remarks

http://www.ral.ucar.edu/projects/icp

• See ICP web page under References and Special Collection for full references from these slides.

• ICP2 starting up! Goal is to investigate precipitation and wind fields over more complex terrain.

• Participation in the ICP is encouraged. Sign up to receive emails at the web site.

• New R software package for image warping is under development.

• New R Software package SpatialVx will contain all of the image warping techniques given here (via the to-be-submitted warping package), and most other techniques of the ICP, as well as others.